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MPIDR Working Paper WP 2024-029 | September 2024
<https://doi.org/10.4054/MPIDR-WP-2024-029>

**Arriaga meets Kitagawa.
Life expectancy decomposition
with population subgroups**

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1 Arriaga meets Kitagawa. Life expectancy
2 decomposition with population subgroups

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15 **Abstract**

16 **Background:** An Arriaga decomposition partitions differences in life
17 expectancy into contributions from mortality rate differences in each age.
18 A Kitagawa decomposition partitions a difference between two weighted
19 means into effects from differences in structure and from differences in
20 each element of the weighted value.

21 **Methods:** Life expectancy differences between like-defined subpopula-
22 tions can be decomposed using the Arriaga method, or a variant thereof.
23 The results of subgroup-specific decompositions can be weighted together
24 using the value component from a Kitagawa decomposition of total life
25 expectancy, given that subpopulations are blended in the radix age. The
26 composition component of the same Kitagawa decomposition gives the
27 effect of subgroup composition on total life expectancy differences.

28 **Results:** Notable properties of the method include: (i) it accommodates
29 any number of subpopulations, (ii) it easily incorporates cause-of-death
30 information, and (iii) composition is considered in the radix age. We apply
31 the method to Spanish cause- and education-specific data.

32 **Conclusions:** This method can further disentangle the effects of mortality
33 and composition differences, helping to explain or clarify paradoxes and
34 secular change. We give both R code and spreadsheet implementations of
35 the method.

36 **Keywords:** Decomposition, Mortality, Cause of death, Population Structure,
37 Mortality Inequalities

38 **Introduction**

39 Population-level differences in life expectancy have been widely analyzed
40 using decomposition methods to understand the impacts of mortality differ-
41 ences in age groups (e.g., [Arriaga, 1984](#); [Pollard, 1988](#)), or [Kitagawa \(1955\)](#)
42 to understand better the impact of compositional differences in social strata
43 ([Kitagawa, 1977](#)). It has long been known among demographers that improve-
44 ments in social welfare have the potential to bring large scale improvements in
45 mortality conditions even net of medical improvements ([Kitagawa & Hauser,](#)
46 [1968](#)). However, very few demographers have ventured to account for both
47 age-specific effects and compositional effects at the same time ([Hendi & Ho,](#)
48 [2021](#); [Su, van Raalte, Aburto, & Canudas-Romo, 2024](#); [Su, Welsh, Korda, &](#)
49 [Canudas-Romo, 2023](#); [Torres, Canudas-Romo, & Oeppen, 2019](#)),. Disentan-
50 gling the compositional effect within a conventional age group decomposition
51 would improve our understanding of life expectancy differences between
52 populations.

53 The life expectancy decomposition method proposed by [Arriaga \(1984\)](#)
54 separates a change in life expectancy into contributions due to mortality
55 changes at different ages. This technique is designed to be practical in that
56 it is framed in terms of lifetable columns expressed in discrete age groups.
57 Two well-known properties of the method are that mortality changes in dif-
58 ferent ages do not need to be proportional and that the derived contributions
59 sum exactly to the observed difference in life expectancy. A documented but

60 lesser-known property of this method is its asymmetry: the absolute val-
61 ues of age-specific contributions depend on whether we compare population
62 one with two, or vice versa. This is why the method is described in terms of
63 changes rather than group differences; the direction of time is clear, so we
64 always decompose an earlier period against a later period. Importantly, the
65 method is designed to work with homogeneous populations, meaning a single
66 lifetable represents each population. Let's call this method and the variant of
67 it that we later apply the "Arriaga method". The Arriaga method has differ-
68 ent extensions available to separate the effects of different causes of death
69 within age groups. We seek to extend this method to account for lifestables
70 that are composed of subpopulations with different mortality.

71 The decomposition method proposed by [Kitagawa \(1955\)](#) separates dif-
72 ferences in a weighted mean into contributions from differences in weights
73 (structure) and the value being weighted. Often, the weighted values are
74 rates, but in our case, the weighted values will be the life expectancies
75 of subgroups. This method is well-known to partition differences owing to
76 "structure" and "value" components, which sum exactly to the observed
77 difference in weighted means. The individual elements of the value being
78 weighted (in our case, life expectancies) have identifiable effects. It is not well-
79 known that the individual elements of the structure component do not have
80 identifiable effects. Instead, the structure effects should be summed up as a
81 total marginal effect due to composition differences. We refer to this method
82 as the "Kitagawa" method.

83 We propose to treat the problem of life expectancy decomposition by age,
84 where a total life expectancy is calculated as the weighted mean of the
85 expectancies of subpopulations. This is only practical when each population
86 is composed of like-defined subpopulations. For example, we may decompose
87 the life expectancy difference between two countries considering differences
88 in education structure, since life expectancy varies by education level ([Mack-
89 enbach et al., 2019](#); [Trias-Llimós, Spijker, Blanes, & Permanyer, 2023](#)). Or,

90 one might wish to calculate a national life expectancy as the weighted average
91 of its regions at two points in time since regional weightings and inequali-
92 ties may have changed over time. In such scenarios, a decomposition should
93 yield the contribution to the overall life expectancy difference stemming from
94 rate differences in each age (and potentially causes of death) within each
95 subpopulation while giving the overall effect of compositional differences. We
96 propose a straightforward analytic method to perform such decompositions.

97 In short, differences in life expectancy between subpopulations can be
98 decomposed using the Arriaga method, one of its variants, or by any other
99 life expectancy decomposition method that is acceptably precise with discrete
100 data (Andreev, Shkolnikov, & Begun, 2002; Horiuchi, Wilmoth, & Pletcher,
101 2008; Ponnappalli, 2005). In a second step, the Kitagawa method is then used
102 to rescale group-specific life expectancy decompositions, yielding an extra
103 component telling us the importance of compositional differences. We justify
104 this second-pass rescaling and demonstrate that the results of this procedure
105 align with those of a linear integral reframing of the problem, per Horiuchi et
106 al. (2008). We then apply these methods to decompose the sex gap between
107 Spanish men and women in remaining life expectancy at age 35, considering
108 differences in composition by education attainment around the same age and
109 considering cause-specific mortality rates.

110 **Method**

111 **Notation**

112 We use the following variables and scripting, most of which are lifetable
113 columns:

- 114 • ${}_n m_a$ mortality rate in the age interval $[a, a + n)$.
- 115 • ℓ_a lifetable survivorship at exact age a .
- 116 • ${}_n L_a$ lifetable person-years lived in the interval $[a, a + n)$.
- 117 • T_a total lifetable person-years lived beyond age a .

- 118 • e_a remaining life expectancy at exact age a .
- 119 • π_a^s the population fraction for subgroup s in age a .

120 We use the superscript s to index subpopulations comprising the total
 121 population, and the superscript “1” to typically indicate the population with
 122 the lower value of e_0 , and “2” for the higher value, such that their difference,
 123 $\Delta = e_0^2 - e_0^1$, is always positive. Causes of death may be indicated with a
 124 c subscript, i.e. $m_{a,c}$. Throughout the manuscript, we assume single-year
 125 age groups and omit age interval (left-side) subscripts (n) where they would
 126 otherwise be due.

127 **Averaging life expectancy**

128 [Vaupel \(2002\)](#) distinguishes between two major approaches to calculate life
 129 expectancy for a total population composed of observed subgroups. The dom-
 130 inant approach is based on averaging the mortality rate in each age by
 131 aggregating deaths and exposures over all subpopulations and calculating
 132 the rate per equation (1):

$$m_a = \frac{\sum_i D_a^i}{\sum_i E_a^i} = \sum_i \pi_a^i m_a^i \quad , \quad (1)$$

133 where D and E stand for deaths and exposures, respectively. This is the stan-
 134 dard method used implicitly or explicitly by national statistical offices, the
 135 Human Mortality Database ([Wilmoth et al., 2021](#)), or the World Population
 136 Prospects ([UN Population Division, 2022](#)), among others. [Vaupel \(2002\)](#) called
 137 this approach the *current rates* perspective, which treats a heterogeneous
 138 population as a homogeneous one.

139 A second approach derives independent subgroup-specific lifetables and
 140 then weights life expectancies together based on an initial mixing composition
 141 to obtain the total life expectancy per equation (2):

$$e_0 = \sum_s \pi_0^s \cdot e_0^s \quad . \quad (2)$$

142 This method aligns with Vaupel's *current conditions* approach, at least to
 143 the extent that heterogeneous conditions are accounted for using observed
 144 strata. This second approach is commonly applied in multistate models with
 145 strata combined according to mixed initial conditions to obtain the total pop-
 146 ulation (Caswell, de Vries, Hartemink, Roth, & van Daalen, 2018; Caswell &
 147 van Daalen, 2021), occasionally also with standard lifetables (Gupta, 1988;
 148 Muszynska-Spielauer & Riffe, 2022), or when calculating between-within
 149 decompositions of variance or other similar summary measures (Riffe, van
 150 Raalte, & Dudel, 2024; Seaman, Riffe, & Caswell, 2019). If the composition
 151 is more favorable in younger than in older ages (for example when younger
 152 generations have higher average educational attainment), the life expectancy
 153 obtained with (2) will be higher than one based on (1). This is the total life
 154 expectancy approach that our proposed method is designed to decompose.

155 **Kitagawa decomposition**

156 Equation (2) treats the total life expectancy as a weighted average, allowing
 157 us to precisely decompose a difference in e_0 , i.e. where $\Delta = e_0^2 - e_0^1$, using the
 158 formulas from Kitagawa (1955). We presume that population 2 has the higher
 159 life expectancy of the two, such that Δ is positive. Specifically, equation (3)
 160 gives an overall effect of differences in composition:

$$\Delta^{(\pi)} = \sum_s (\pi^{s,2} - \pi^{s,1}) \cdot \bar{e}_0^s \quad , \quad (3)$$

161 where

$$\bar{e}_0^s = \frac{e_0^{s,1} + e_0^{s,2}}{2} \quad (4)$$

162 This result is widely known. Note that the the composition effect must be
 163 summed this way because group-specific composition (structure) effects are
 164 not well-identified and therefore cannot be interpreted in isolation. Equation
 165 (5) gives the subgroup-specific effects of differences in life expectancy:

$$\Delta^{(e_0^s)} = (e_0^{s,2} - e_0^{s,1}) \cdot \bar{\pi}^s \quad , \quad (5)$$

166 where $\bar{\pi}^s$ is the average composition, analogous to equation (4), and where the
 167 superscripts are consistent with those used to calculate the total difference
 168 Δ .

169 Equation (6) states that the observed difference in total life expectancy
 170 is the sum of (i) a single component capturing the effect of compositional
 171 change and (ii) a set of components giving the contribution of each subgroup's
 172 life expectancy difference to the total life expectancy difference. This second
 173 effect could equivalently be called the *rate* effect because each life expectancy
 174 is ultimately a function of mortality rates.

$$\Delta = \Delta^{(\pi)} + \sum_s \Delta^{(e_0^s)}. \quad (6)$$

175 **Symmetrical Arriaga decomposition**

176 The *rate* effect as per (5) can be viewed as the *net* effect on total life expectancy
 177 differences resulting from the age-specific (potentially also cause-specific)
 178 differences between like subgroups, as isolated by various life expectancy
 179 decomposition methods. In this setting, the choice of method to derive age-
 180 specific effects for subgroup-specific changes in life expectancy is not crucial.
 181 The [Arriaga \(1984\)](#) decomposition approach is widely favored because it
 182 is designed for discrete data, it is analytic (implying computational effi-
 183 ciency), and because decomposition results sum exactly to the observed life
 184 expectancy difference. Since our application concerns sex differences rather
 185 than changes over time, we consider a *symmetrical* Arriaga decomposition.
 186 Since we also include cause-of-death information, we transform the symmet-
 187 rical decomposition results into an implied sensitivity function. This exercise
 188 is repeated for each subgroup to decompose group-specific changes in life
 189 expectancy (indicated with the superscript s on each variable):

$$\Delta^s = e_0^{s,2} - e_0^{s,1} \quad (7)$$

190 In this equation Δ^s is the subgroup-specific (s) difference in life expectancy
 191 being decomposed, which is composed of age-specific contributions, $\overrightarrow{\Delta}_a^s$ or
 192 $\overleftarrow{\Delta}_a^s$ depending on whether we decompose from population 1 to 2 (forward) or
 193 from 2 to 1 (backward). The forward age-specific values $\overrightarrow{\Delta}_a^s$ can be calculated
 194 following Arriaga's decomposition method, consistent with [Arriaga \(1984\)](#) or
 195 the presentation in [Preston, Heuveline, and Guillot \(2000\)](#), as outlined in
 196 equation (8). We use a life table radix of 1, meaning $\ell_0 = 1$, to simplify the
 197 formula slightly.

$$\overrightarrow{\Delta}_a^s = \begin{cases} \ell_a^{s,1} \cdot \left(\frac{I_a^{s,2}}{\ell_a^{s,2}} - \frac{I_a^{s,1}}{\ell_a^{s,1}} \right) + T_{a+n}^{s,2} \cdot \left(\frac{\ell_a^{s,1}}{\ell_a^{s,2}} - \frac{\ell_{a+n}^{s,1}}{\ell_{a+n}^{s,2}} \right) & \forall a < \omega, \\ \ell_\omega^{s,1} \cdot (e_\omega^{s,2} - e_\omega^{s,1}) & \forall a = \omega. \end{cases} \quad (8)$$

198 Equation (8) represents the first pass of our symmetrical decomposition,
 199 while (9) is the second pass, which is identical except for swapping population
 200 superscripts.

$$\overleftarrow{\Delta}_a^s = \begin{cases} \ell_a^{s,2} \cdot \left(\frac{I_a^{s,1}}{\ell_a^{s,1}} - \frac{I_a^{s,2}}{\ell_a^{s,2}} \right) + T_{a+n}^{s,1} \cdot \left(\frac{\ell_a^{s,2}}{\ell_a^{s,1}} - \frac{\ell_{a+n}^{s,2}}{\ell_{a+n}^{s,1}} \right) & \forall a < \omega, \\ \ell_\omega^{s,2} \cdot (e_\omega^{s,1} - e_\omega^{s,2}) & \forall a = \omega. \end{cases} \quad (9)$$

201 Importantly,

$$\begin{aligned} -\Delta^s &= \sum_a \overleftarrow{\Delta}_a^s \\ &= e_0^1 - e_0^2. \end{aligned} \quad (10)$$

202 A symmetrical estimate of Δ_a^s is given by the sign-adjusted average of (8)
 203 and (9) for each age, again with a preference for a positive difference:

$$\Delta_a^s = \frac{(\overrightarrow{\Delta}_a^s - \overleftarrow{\Delta}_a^s)}{2}. \quad (11)$$

204 Repeat the symmetrical Arriaga steps to derive age-specific contributions,
 205 Δ_a^s , for each life expectancy difference between like-defined subgroups. To

206 take into account information on causes of death, it is best to derive an ad-
 207 hoc all-cause sensitivity function, s_a^s , by dividing the decomposition result by
 208 the mortality rate difference:

$$s_a^s = \frac{\Delta_a^s}{m_a^{s,2} - m_a^{s,1}} \quad , \quad (12)$$

209 and then multiply age-cause-specific mortality rate differences into s_a^s
 210 to obtain the subgroup-specific decomposition by age and cause per
 211 equation (13).

$$\Delta_{a,c}^s = s_a^s \cdot (m_{a,c}^{s,2} - m_{a,c}^{s,1}) \quad . \quad (13)$$

212 This way of accounting for causes of death is exactly additive. However, if
 213 any all-cause mortality rate difference is close to 0, equation (12) is vulnera-
 214 ble, in which case we advise deriving the sensitivity using a more robust (but
 215 less precise) approach (Riffe & Atance, 2024)

216 Rescale Arriaga results

217 To obtain the *net* impact of Δ_a^s (or $\Delta_{a,c}^s$) on overall life expectancy change we
 218 rescale using (14) to match the life expectancy components from equation (5).

$$\Delta_{a,c}^{s,\text{net}} = \Delta^{e_0^s} \cdot \frac{\Delta_{a,c}^s}{\Delta^s} \quad , \quad (14)$$

219 With the composition effect from equation (3) (Δ^π) and (14), we have all
 220 elements of the proposed decomposition:

$$\Delta = \Delta^\pi + \sum_s \sum_a \sum_c \Delta_{a,c}^{s,\text{net}} \quad . \quad (15)$$

221 Application

222 We use data from Trias-Llimós et al. (2023) on sex-, age-, education-,
 223 and cause-specific death counts for individuals aged 35 and over for the
 224 years 2016-21 in Spain, which were obtained by request from the Span-
 225 ish National Statistics Institute (INE). We use all-cause mortality rates for

226 each education- and sex-specific subpopulation from the same source. Edu-
 227 cational attainment information is not recorded on death certificates, but
 228 INE adds this variable through multiple data linkages [INE \(2020\)](#), includ-
 229 ing municipal population registers (“Padrón”) and the 2011 census. We
 230 redistributed deaths with missing education codes ($< 2\%$ of deaths) across
 231 the four educational groups within age, sex, and cause of death classes,
 232 proportional to deaths with non-missing education. We then grouped edu-
 233 cational attainment into three categories: Low (primary education or less),
 234 Medium (secondary education), and High (tertiary or university education).
 235 We grouped causes of death from the original ICD10 4-digit codes into 17
 236 main causes: Infectious (ICD-10 codes: A00-B99), Neoplasms (C00-D48),
 237 Blood (D50-D89), Endocrine (E00-E90), Mental (F00-F99), Nervous (G00-
 238 G99), Circulatory (I00-I99), Respiratory (J00-J99), Digestive (K00-K93), Skin
 239 (L00-L99), Musculoskeletal (M00-M99), Genitourinary (N00-N99), Congeni-
 240 tal (Q00-Q99), Ill-defined (R00-R94), External (chapters S, T, V, W, X and Y),
 241 Other causes (chapters H, O and P), and COVID-19 (U071-U072). Moreover,
 242 to separate the pandemic period we combined years into two time periods:
 243 2016-2019 and 2020+. Our main results refer to 2016-2019, whereas extra
 244 results for 2020-2021 are given in the materials repository. We smoothed
 245 and graduated the original data from 5-year age groups to single ages using
 246 a generalized additive model (GAM), including a P-spline over age and pop-
 247 ulation offsets ([S. Wood, 2017](#); [S.N. Wood, 2011](#)). The GAM formula is given
 248 in equation (16), using the Quasi-Poisson family of models to account for
 249 possible overdispersion or Negative Binomial model in cases with a more
 250 substantial overdispersion:

$$\log(\mu_a) = \beta_0 + f(a) + \ln(pop_a). \quad (16)$$

251 **Results**

252 We present empirical results for the decomposition of the sex difference in
 253 life expectancy in 2016-2019. Table 1 presents the components of the Kita-
 254 gawa decomposition for males and females. In this context, π^c represents
 255 the educational structure, while $e(35)$ denotes life expectancy at age 35 (“rate”
 256 element in original Kitagawa terms). The table also gives the corresponding
 257 education structure and life expectancy differences between sexes. The final
 258 element, $\Delta^{(e_{35}^s)}$, is the decomposition result, which gives the education group
 259 contribution. The largest life expectancy gap is observed among individuals
 260 with Primary and Secondary education levels, and this gap is lower for those
 261 with Higher education.

Education	π^f	π^m	$\pi^f - \pi^m$	$e(35)^f$	$e(35)^m$	$\overline{e(35)}$	$\frac{e(35)^f - e(35)^m}{e(35)^m}$	$\Delta^{(e_{35}^s)}$
Primary	0.09	0.10	-0.01	48.21	42.47	45.34	5.74	0.54
Secondary	0.41	0.52	-0.11	50.29	44.52	47.40	5.78	2.69
Higher	0.50	0.38	0.12	51.30	46.92	49.11	4.38	1.92

Table 1 Elements of the Kitagawa decomposition, sex gaps in life expectancy in Spain, 2016-19

262 The corresponding difference in education-specific life expectancy is
 263 also illustrated in Figure 1. This figure additionally shows the differences
 264 observed for standard aggregated (non-stationary) versus radix-weighted life
 265 expectancy (stationary).

266 Figure 2 is complementary to Figure 1 and represents the education con-
 267 tribution to the $e(35)$ sex gap with the educational composition component
 268 contribution, about 3 months, being added to the figure.

269 Figure 3 illustrates the cause-specific contributions to the life expectancy
 270 gap. The majority of this gap is explained by differences in mortality from
 271 cancer, circulatory diseases, respiratory conditions, and external causes.
 272 Neoplasms alone contribute to almost two full years of life expectancy.

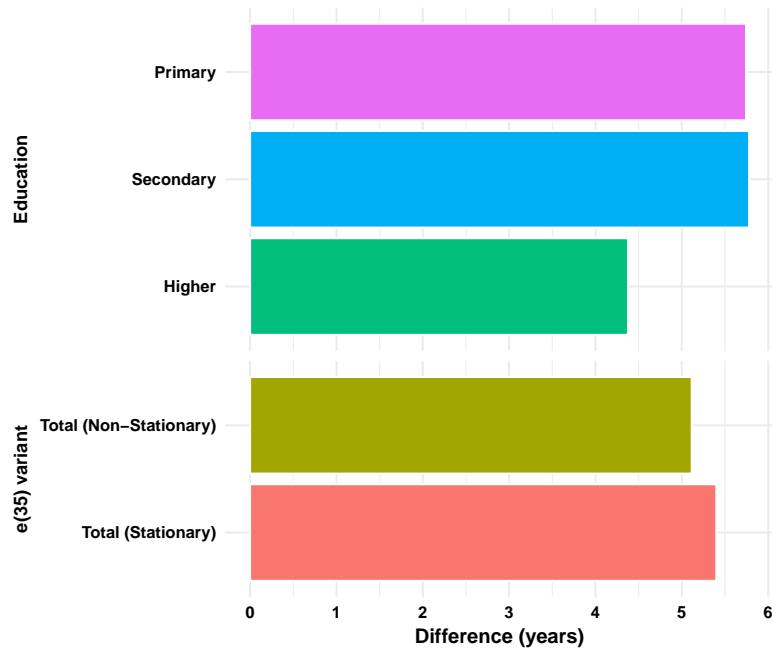


Fig. 1 The Female-Male difference in $e(35)$ by education and population type, Spain 2016-19.

273 The only categories where males have a slight advantage in mortality are
 274 musculoskeletal and other minor causes.

275 Further decomposition of cause-specific differences for age and education
 276 components of composed decomposition is shown in Figure 4. The majority
 277 of the difference in neoplasm mortality is concentrated in the older age group
 278 of 70-80 years. The circulatory component is observed in slightly younger
 279 ages and is more uniformly distributed between ages 60 and 80. In contrast,
 280 the respiratory component is more pronounced around age 85. The external
 281 causes of death primarily contribute to the difference in younger ages and
 282 since our data is bounded below by the age of 35, the full effect cannot be
 283 fully observed here.

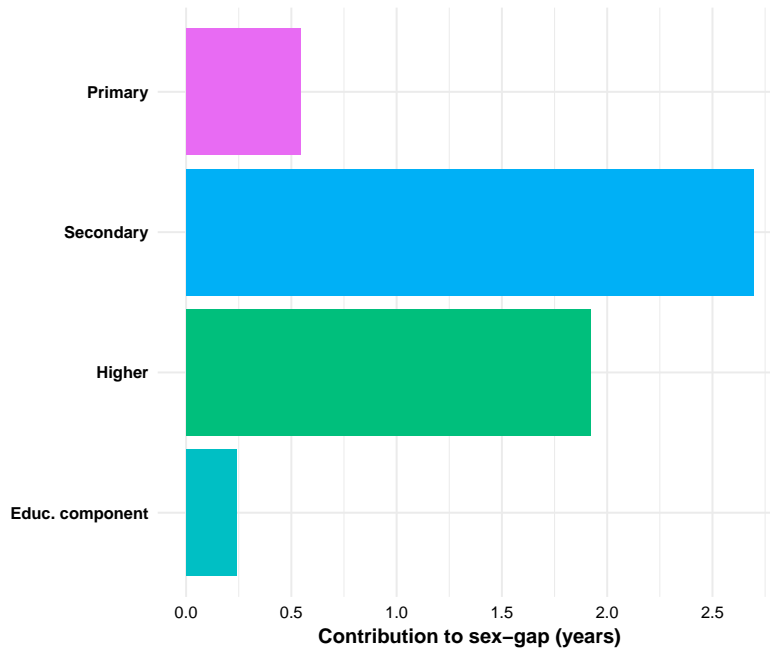


Fig. 2 Education structure contribution to total $e(35)$ sex gap, Spain 2016-19

284 Discussion

285 In this paper, we propose a method for decomposing the differences in life
 286 expectancy that accounts for the compositional effects of education and
 287 causes of death between two populations mixed in the radix. Deriving an
 288 analytical solution for decomposition offers several advantages over alterna-
 289 tive methods, particularly in terms of simplicity, speed, and repeatability.
 290 Unlike the linear integral method [Horiuchi et al. \(2008\)](#), which requires pro-
 291 gramming expertise, our approach can be executed swiftly and efficiently,
 292 even within spreadsheet-like environments (see reproducibility repository
 293 for an example). The computational efficiency of our method allows for the
 294 calculation of confidence intervals using bootstrapping.

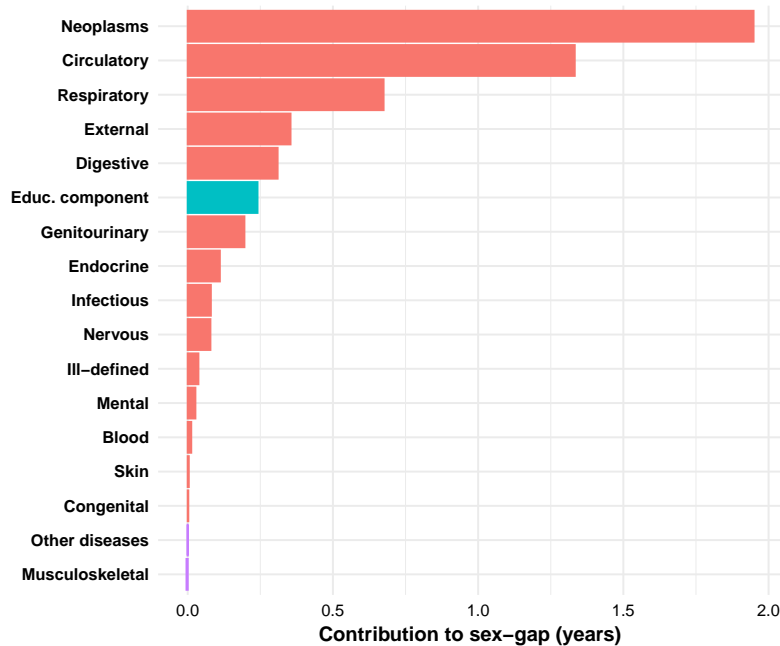


Fig. 3 Cause-specific contribution to sex difference in total $e(35)$, Spain 2016-19. The Education structure contribution is presented in a different colour

295 Using the Kitagawa decomposition approach (Kitagawa, 1955) to weigh
 296 together group-specific paired decompositions, our method can be general-
 297 ized to address many decomposition problems involving structural composi-
 298 tional components, and it is not bound to mortality analyses. However, this
 299 post-weighting approach is most suitable for cases where populations are
 300 blended in a radix. In contrast, other approaches in the literature (Hendi
 301 & Ho, 2021; Su et al., 2024, 2023; Torres et al., 2019) decompose refer-
 302 ring to the standard *current rates* (Vaupel, 2002) method of calculating life
 303 expectancy. These approaches treat the age pattern of group prevalence dif-
 304 ferently, in essence fixing group prevalence (weights) rather than making
 305 prevalence depend on mortality. We point out that mortality is often one of
 306 the major drivers of how group prevalence weights change over age.

307 When incorporating information on causes of death, a potential vulner-
 308 ability of our method arises when the difference in mortality rates in the

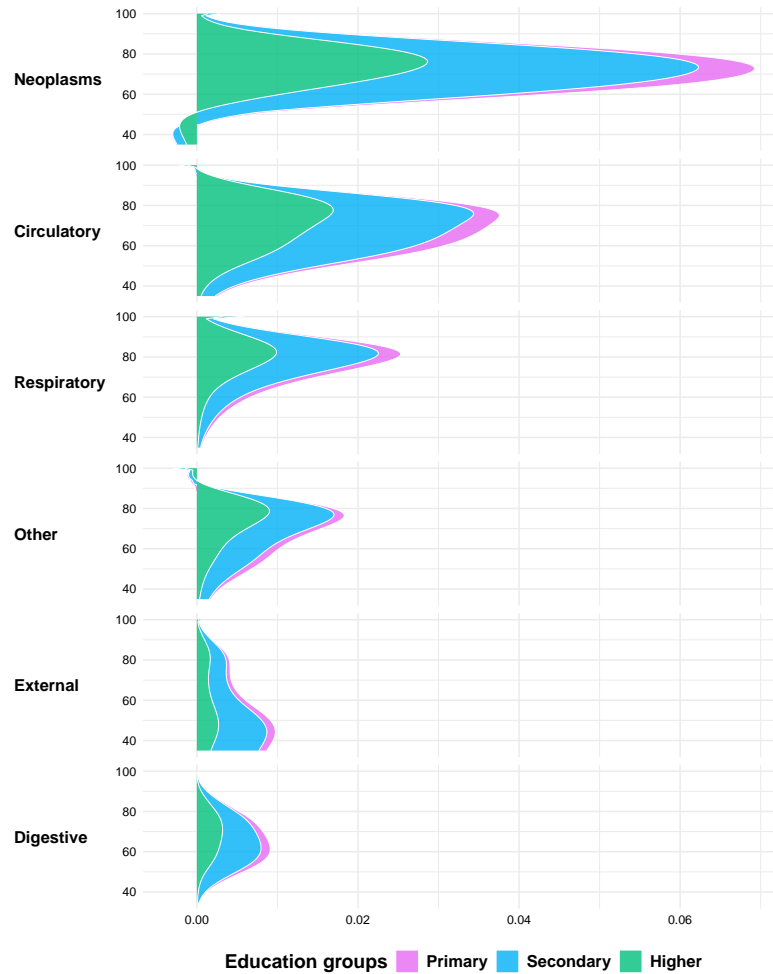


Fig. 4 The contributions of age, education, and major causes of death to the sex gap in total $e(35)$.

309 denominator of equation (12) approaches zero (compare with Box 4.3 of
 310 [Preston et al. \(2000\)](#)), rendering an implausible result. In this case, one
 311 may prefer to use a direct approximation or numerical estimate of the life
 312 expectancy sensitivity in equation (13).

313 Our empirical results indicate that the sex gap in life expectancy remains
 314 a significant issue in Spain across all educational strata. However, it is about
 315 25% lower in the Higher education group. This can be attributed to several

316 factors, including females' lower engagement in risky behaviours (Byrnes,
317 Miller, & Schafer, 1999; Cook & Bellis, 2001; Kritsotakis, Psarrou, Vassi-
318 laki, Androulaki, & Philalithis, 2016; Olson, Hummer, & Harris, 2017; Ross,
319 Masters, & Hummer, 2012), better jobs and greater health awareness of
320 males with higher education (Lawrence, 2017; McMahon, 2009; Ross et al.,
321 2012; Ross & Wu, 1995). In terms of causes, neoplasms, circulatory dis-
322 eases, and respiratory diseases account for the largest contributions to the
323 sex gap. Given that the male disadvantage peaks around ages 65-85 (birth
324 cohorts 1934-1951), this difference can be partly explained by the differences
325 in smoking patterns between males and females within each subpopulation
326 (Haeberer et al., 2020). Since our findings are based on education as a marker
327 of socioeconomic status, they refer only to ages above 35. Therefore, we do
328 not here measure the full power of socioeconomic status or different causes
329 of death in driving the overall sex gap in life expectancy. Further studies
330 could use our method to explain the sex gap using geography, socioeconomic
331 status, and causes of death.

332 Our proposed combination of two analytical decomposition methods can
333 handle the problem of a composed life expectancy, thereby giving an efficient
334 framework for analyzing differences in life expectancy while accounting for
335 population heterogeneity on fixed characteristics. We give both R code and
336 spreadsheet implementations of the method. Our framework is straightfor-
337 ward to use and does not require high computational capacity, spreadsheet
338 implementation in a spreadsheet-like environment while providing results
339 comparable to those of the widely used Horiuchi method. The main advan-
340 tages of the proposed method are its computational efficiency and ease of
341 implementation over the commonly used Horiuchi method. Further efforts
342 in alleviating the sex gap in mortality are required. The main cause of death
343 contributions suggests that attention to the determinants of neoplasms,
344 cardiovascular, and respiratory diseases could play a substantial role in
345 reducing the sex gap in life expectancy.

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490
491 **Acknowledgments.** Thanks to Michael Lachanski for posing the question
492 that led to the solution we present here, to Jonas Schoeley for first raising
493 our attention to the composition problem with the Kitagawa method, and to
494 the MPIDR Laboratory of Population Health for posing assorted questions in
495 September 2023 that also led to this work.

496 **Funding**

497 TR acknowledges funding from *la Caixa Foundation* grant nr SR22-00502
498 from and from Spanish *Ministerio de Ciencia e Innovation* grant num-
499 ber PID2022-142762OA-I00. STL acknowledges research funding from the
500 Ramon y Cajal program of the same Ministry (RYC2021-033123-I).

501 **Conflict of interest**

502 The authors declare no conflict of interest

503 **Availability of data and materials**

504 We cannot share the original data that this study is based on, but these can
505 be requested either from the Spanish INE (<https://www.ine.es>) or from [Trias-](#)
506 [Llimós et al. \(2023\)](#) for purposes of reproducing our results from the source
507 data stage. We share smoothed and graduated single-age mortality rates nec-
508 essary to reproduce all our results in an open OSF repository (anonymized for
509 peer review)

510 https://osf.io/xnmv6/?view_only=047c4f8fe11c478a9aa71e8993b73e22

511 **Code availability**

512 All code for this paper is available in an open OSF repository
513 (anonymized for peer review) [https://osf.io/xnmv6/?view_only=](https://osf.io/xnmv6/?view_only=047c4f8fe11c478a9aa71e8993b73e22)

514 [047c4f8fe11c478a9aa71e8993b73e22](https://osf.io/xnmv6/?view_only=047c4f8fe11c478a9aa71e8993b73e22)